SAMPLE PAPER: MATHEMATICS

CLASS–XII: 2014–15

TYPOLOGY

	VSA (1 M)	LA–I (4 M)	LA–II (6 M)	100
Remembering	2, 5	11, 15, 19	24	20
Understanding	1, 4	8, 12	23	16
Applications	6	14, 18, 13	21, 26	25
HOTS	3	10, 17	20, 22	21
Evaluation & MD	–	7, 9, 16	25	18

SECTION–A

Question number 1 to 6 carry 1 mark each.

1. The position vectors of points A and B are $\vec{a}$ and $\vec{b}$ respectively.

   P divides AB in the ratio 3 : 1 and Q is mid-point of AP. Find the position vector of Q. 1

2. Find the area of the parallelogram, whose diagonals are $\vec{d}_1=5\hat{\imath}$ and $\vec{d}_2=2\hat{\jmath}$ 1

3. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane. 1

4. If $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$, Write the cofactor of a_{32} (the element of third row and 2nd column). 1

5. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3 \left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m+n. 1

6. Write the differential equation representing the curve $y^2 = 4ax$, where a is an arbitrary constant. 1

SECTION-B

Question numbers 7 to 19 carry 4 marks each.

7. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at the rate of Rs. 20, Rs.15 and Rs. 5 per unit respectively. School A sold 25 paper-bags 12 scrap-books and 34 pastel sheets. School B sold 22 paper-bags, 15 scrapbooks and 28 pastel-sheets while school C sold 26 paper-bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school.

   By such exhibition, which values are inculcated in the students? 4

8. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$, then show that $A^2 - 4A + 7I = O$.

Using this result calculate A^3 also.

OR

If A = $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix}$, find A^{-1} , using elementary row operations. 4

9. If x, y, z are in GP, then using properties of determinants, show that

$\begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix}$ = o, where $x \neq$ y $\neq$ z and p is any real number. 4

10. Evaluate : $\int_{-1}^{1} |x\, cos\pi\, x| \mathrm{d}x$. 4

11. Evaluate : $\int \frac{1+sin2x}{1+cos2x}$. e^{2x} dx. 4

OR

Evaluate : $\int \frac{x^4}{(x-1)(x^2+1)}$ dx

12. Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 3' given that 'there is at least one head'. 4

OR

How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

13. For three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ if $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \times \vec{c} = \vec{b}$, then prove that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, $|\vec{b}| = |\vec{a}|$ and $|\vec{a}| = 1$ 4

14. Find the equation of the line through the point (1,-1,1) and perpendicular to the lines joining the points (4,3,2), (1,-1,0) and (1,2,-1), (2,1,1) 4

OR

Find the position vector of the foot of perpendicular drawn from the point P(1,8,4) to the line joining A(O,-1,3) and B(5,4,4). Also find the length of this perpendicular.

15. Solve for x: $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$

OR

Prove that: $2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$ 4

16. If $x = \sin t$, $y = \sin kt$, show that

$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + k^2y = 0$ 4

17. If $y^x + x^y + x^x = a^b$, find $\frac{dy}{dx}$ 4

18. It is given that for the function $f(x) = x^3 + bx^2 + ax + 5$ on [1, 3], Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$.

Find values of a and b. 4

19. Evaluate : $\int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$ 4

SECTION-C

Question numbers 20 to 26 carry 6 marks each.

20. Let A = {1, 2, 3, ..., 9} and R be the relation in A x A defined by (a, b) R (c, d) if a+d = b+c for a, b, c, d ∈ A.

Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)]. 6

OR

Let $f : N \longrightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$.

Show that $f : N \longrightarrow S$ is invertible, where S is the range of f. Hence find inverse of f.

21. Compute, using integration, the area bounded by the lines

$x+2y = 2$, $y-x=1$ and $2x+y= 7$ 6

22. Find the particular solution of the differential equation 6

$xe^{\frac{y}{x}} - y\sin\left(\frac{y}{x}\right) + x\frac{dy}{dx}\, sin\left(\frac{y}{x}\right) = 0$, given that

y = 0, when x = 1

OR

Obtain the differential equation of all circles of radius *r*.

23. Show that the lines $\vec{r} = (-3\hat{\imath} + \hat{\jmath} + 5\hat{k}) + \lambda\,(-3\hat{\imath} + \hat{\jmath} + 5\hat{k})$ and $\vec{r} = \left(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right) + \mu\left(-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right)$ are coplanar. Also, find the equation of the plane containing these lines. 6

24. 40% students of a college reside in hostel and the remaining reside outside. At the end of year, 50% of the hosteliers got A grade while from outside students, only 30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hostelier? 6

25. A man rides his motorcycle at the speed of 50km/h. He has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 80km/h, the petrol cost increases to Rs. 3 per km. He has atmost Rs. 120 to spend on petrol and one hour's time. Using LPP find the maximum distance he can travel. 6

26. A jet of enemy is flying along the curve y = x^2+2 and a soldier is placed at the point (3, 2). Find the minimum distance between the soldier and the jet. 6

MARKING SCHEME

SAMPLE PAPER

SECTION-A

1.	$\frac{1}{8}\left(5\vec{a} + 3\vec{b}\right)$	1
2.	5 sq. units	1
3.	$\vec{r}.\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right) = 29$	1
4.	-14	1
5.	m + n = 4	1
6.	$2x\frac{dy}{dx} - y = 0$	1

SECTION-B

7. Sale matrix for A, B and C is $\begin{pmatrix} 25 & 12 & 34 \\ 22 & 15 & 28 \\ 26 & 18 & 36 \end{pmatrix}$ ½

Price matrix is $\begin{pmatrix} 20 \\ 15 \\ 5 \end{pmatrix}$ ½

$\therefore \quad \begin{pmatrix} 25 & 12 & 34 \\ 22 & 15 & 28 \\ 26 & 18 & 36 \end{pmatrix}\begin{pmatrix} 20 \\ 15 \\ 5 \end{pmatrix} = \begin{pmatrix} 500+ & 180+ & 170 \\ 440+ & 225+ & 140 \\ 520+ & 270+ & 180 \end{pmatrix}$ ½

$\therefore$ Amount raised by $= \begin{pmatrix} 850 \\ 805 \\ 970 \end{pmatrix}$ ½

School A = Rs 850, school B = Rs 805, school C = Rs 970

Values

- Helping the orphans 1
- Use of recycled paper 1

8. $A^2 = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 12 \\ -4 & 1 \end{pmatrix}$ 1

$\therefore A^2 - 4A + 7I = \begin{pmatrix} 1 & 12 \\ -4 & 1 \end{pmatrix} + \begin{pmatrix} -8 & -12 \\ 4 & -8 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ 2

$A^2 = 4A - 7I \Rightarrow A^3 = 4A^2 - 7A = 4(4A-7I) - 7A$

$= 9A - 28I = \begin{pmatrix} 18 & 27 \\ -9 & 18 \end{pmatrix} + \begin{pmatrix} -28 & 0 \\ 0 & -28 \end{pmatrix}$

$= \begin{pmatrix} -10 & 27 \\ -9 & -10 \end{pmatrix}$ 1

OR

Write A = IA we get $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.A$ ½

$R_2 \longrightarrow R_2 - 2R_1 \Rightarrow$ $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_2 \longrightarrow R_2 - 3R_3 \Rightarrow$ $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_1 \longrightarrow R_1 + R_2 \Rightarrow$
$R_3 \longrightarrow R_3 - 2R_2$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix} A$ 1

$\therefore A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix}$ ½

9. $\Delta = \begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix}$

$C_1 \longrightarrow C_1 - pC_2 - C_3$, $\Delta = \begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -p^2x - py - py - z & px+y & py+z \end{vmatrix}$ 1½

Expanding by R_3

$\Delta = (-p^2x - 2py - z)(xz - y^2)$ 1

Since x, y, z are in GP, $\therefore$ $y^2 = xz$ or $y^2 - xz = 0$ 1

$\therefore \quad \Delta = 0$ ½

10. $\int_{-1}^{1}|x.\cos\pi x|\,dx = 2\int_0^1|x\cos\pi x|dx$ 1

$= 2\int_0^{½}(x\cos\pi x)\,dx + 2\int_{½}^{1} -(x\cos\pi x)dx$ 1

$= 2\left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_0^{\frac{1}{2}} - 2\left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{\frac{1}{2}}^{1}$ 1

$= 2\left[\frac{1}{2\pi} - \frac{1}{\pi^2}\right] - 2\left[\frac{-1}{\pi^2} - \frac{1}{2\pi}\right] = \frac{2}{\pi}$ 1

11. $I = \int \frac{1+\sin 2x}{1+\cos 2x} . e^{2x}\,dx = \frac{1}{2}\int \frac{1+\sin t}{1+\cos} . e^t\,dt$ (where $2x = t$) ½

$= \frac{1}{2}\int\left(\frac{1}{2\cos^2 t/2} + \frac{2\sin t/2\ \cos t/2}{2\cos^2 t/2}\right)e^t\,dt$ 1

$= \frac{1}{2}\int\left(\frac{1}{2}\sec^2 t/2 + \tan t/2\right)e^t\,dt$ 1

$\tan t/2 = f(t)$ then $f'(t) = \frac{1}{2}\sec^2 t/2$

Using $\int(f(t) + f'(t))\,e^t\,dt = f(t)\,e^t + C$, we get ½

$I = \frac{1}{2}\tan t/2 . e^t + C = \frac{1}{2}\tan x . e^{2x} + C$ 1

OR

We have

$$\frac{x^4}{(x-1)(x^2+1)} = (x+1) + \frac{1}{x^3 - x^2 + x - 1}$$

$$= (x+1) + \frac{1}{(x-1)(x^2+1)} \quad \text{........ (1)}$$ 1

Now express $\frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)}$ .... (2)

So,

$$1 = A(x^2\ + 1) + (Bx + C)\ (x - 1)$$

$$= (A + B)x^2 + (C - B)x + A - C$$

Equating coefficients, $A + B = 0$, $C – B = 0$ and $A – C = 1$,

Which give $A = \frac{1}{2}$, $B = C = -\frac{1}{2}$. Substituting values of A, B, and C in (2), we get

$$\frac{1}{(x-1)(x^2+1)} = \frac{1}{2(x-1)} - \frac{1}{2}\frac{x}{(x^2+1)} - \frac{1}{2(x^2+1)} \quad \ldots\ldots\ldots (3)$$ 1

Again, substituting (3) in (1), we have

$$\frac{x^4}{(x-1)(x^2+1)} = (x\ +\ 1) + \frac{1}{2(x-1)} - \frac{1}{2}\frac{x}{(x^2+1)} - \frac{1}{2(x^2+1)}$$

Therefore

$$\int \frac{x^4}{(x-1)\,(x^2+1)}\,dx = \frac{x^2}{2} + x + \frac{1}{2}\log\,|x - 1| - \frac{1}{4}\log(x^2 + 1) - \frac{1}{2}tan^{-1}\,x + C$$ 1+1

12. Let E : Die shows a number > 3

E : {H4, H5, H6] ½

and F : there is atleast one head.

∴ F : {HT, H1, H2, H3, H4, H5, H6} ½

$P(F) = 1 – \frac{1}{4} = \frac{3}{4}$ 1

$P(E \cap F) = \frac{3}{12} = \frac{1}{4}$ 1

$\therefore P(E/F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$ 1

OR

$p = \frac{1}{2}$, $q = \frac{1}{2}$, let the coin be tossed n times

$\therefore P(r \geq 1) > \frac{90}{100}$ ½

or $1 - P(r=0) > \frac{90}{100}$ ½

$P(r=0) < 1 - \frac{9}{10} = \frac{1}{10}$ ½

${}^nC_0 \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^0 < \frac{1}{10} \Rightarrow \frac{1}{2^n} < \frac{1}{10}$ 1½

$\Rightarrow 2^n > 10, \therefore n = 4$ 1

13. $\left.\begin{array}{l}\vec{a} \times \vec{b} = \vec{c} \Rightarrow \vec{a} \perp \vec{b} \text{ and } \vec{b} \perp \vec{c} \\ \vec{a} \times \vec{c} = \vec{b} \Rightarrow \vec{a} \perp \vec{b} \text{ and } \vec{c} \perp \vec{b}\end{array}\right\} \Rightarrow \vec{a} \perp \vec{b} \perp \vec{c}$ 1

$|\vec{a} \times \vec{b}| = |\vec{c}|$ and $|\vec{a} \times \vec{c}| = |\vec{b}|$ 1

$\Rightarrow |\vec{a}|\,|\vec{b}| \sin\frac{\pi}{2} = |\vec{c}|$ and $|\vec{a}|\,|\vec{c}| \sin {}^{\pi}/_{2} = |\vec{b}|$

$\Rightarrow |\vec{a}|\,|\vec{b}| = |\vec{c}| \therefore |\vec{a}|\,|\vec{a}|\,|\vec{b}| = |\vec{b}| \Rightarrow |\vec{a}|^2 = 1 \Rightarrow |\vec{a}| = 1$ 1

$\Rightarrow 1.\,|\vec{b}| = |\vec{c}| \Rightarrow |\vec{b}| = |\vec{c}|$

14. DR's of line (L_1) joining (4, 3, 2) and (1, -1, 0) are <3, 4, 2> ½

DR's of line (L_2) joining (1, 2, -1) and (2, 1, 1) are <1, -1, 2> ½

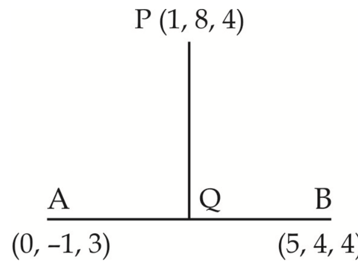
A vector $\perp$ to L_1 and L_2 is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 1 & -1 & 2 \end{vmatrix} = 10\hat{i} - 4\hat{j} - 7\hat{k}$ 1½

$\therefore$ Equation of the line passing through (1, -1, 1) and $\perp$ to L_1 and L_2 is

$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda\,(10\hat{i} - 4\hat{j} - 7\hat{k})$ 1½

OR

Equation of line AB is



$\vec{r}$ = (-$\hat{j}$+3$\hat{k}$) + λ (5$\hat{i}$+5$\hat{j}$+$\hat{k}$) 1

∴ Point Q is (5λ, -1+5λ, 3+λ) ½

$\overrightarrow{PQ}$ = (5λ-1) $\hat{i}$ +(5λ -9) $\hat{j}$ + (λ-1) $\hat{k}$ ½

PQ ⊥ AB ⟹ 5(5λ-1) + 5 (5λ-9) + 1 (λ-1) = 0

51λ = 51 ⟹ λ=1 ½

⟹ foot of perpendicular (Q) is (5, 4, 4) ½

Length of perpendicular PQ = $\sqrt{4^2 + (-4)^2 + 0^2} = 4\sqrt{2}$ units 1

15. $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}\, x = -\frac{\pi}{2}$

⟹ $\sin^{-1} 6x = \left(\frac{-\pi}{2} - \sin^{-1} 6\sqrt{3x}\right)$ ½

⟹ $6x = \sin\left[-\frac{\pi}{2} - \sin^{-1} 6\sqrt{3}x\right] = -\sin\left[\frac{\pi}{2} + \sin^{-1} 6\sqrt{3}x\right]$ ½

$= -\cos\left[\sin^{-1} 6\sqrt{3}x\right] = -\sqrt{1 - 108x^2}$ 1

⟹ $36x^2 = 1 - 108x^2$ ⟹ $144x^2 = 1$

⟹ $x = \pm \frac{1}{12}$

since $x = \frac{1}{12}$ does not satisfy the given equation

∴ $x = -\frac{1}{12}$ 1

OR

LHS = $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$

$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$ 1

$= \tan^{-1}\left(\frac{2.\frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\frac{17}{31}$ 1

$= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\frac{17}{31}$ 1

$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}.\frac{17}{31}}\right) = \tan^{-1}(1) = \pi/4$ 1

16. $x = \sin t$ and $y = \sin kt$

$\frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = k \cos kt$

$\Rightarrow \frac{dy}{dx} = k\,\frac{\cos kt}{\cos t}$ 1

or $\cos t.\frac{dy}{dx} = k.\cos kt$

$\cos^2 t\left(\frac{dy}{dx}\right)^2 = k^2\cos^2 kt$

$\cos^2 t\left(\frac{dy}{dx}\right)^2 = k^2\cos^2 kt$ ½

$(1-x^2)\left(\frac{dy}{dx}\right)^2 = k^2(1-y^2)$ 1

Differentiating w.r.t. x

$(1-x^2)\, 2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = -2k^2y\frac{dy}{dx}$ 1

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2y = 0$ ½

17. let $u = y^x$, $v = x^y$, $w = x^x$

(i) $\log u = x \log y \Rightarrow \frac{du}{dx} = y^x\left[\log y + \frac{x}{y}\frac{dy}{dx}\right]$ 1

(ii) $\log v = y \log x \Rightarrow \frac{dv}{dx} = x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right]$ ½

(iii) log w = x log$x$ $\Rightarrow \frac{dw}{dx}$ = x^x , (1+log$x$) ½

$\Rightarrow y^x \left[\log y + \frac{x}{y}\frac{dy}{dx}\right] + x^y \left[\frac{y}{x} + \log x \frac{dy}{dx}\right] + x^x (1+\log x) = 0$ 1

$\Rightarrow \frac{dy}{dx} = -\frac{x^x(1+\log x)+y\, x^{y-1}+ y^x \log y}{x.y^{x-1} + \log x}$ 1

18. $f(x) = x^3 + bx^2 + ax + 5$ on [1, 3]

$f'(x) = 3x^2 + 2bx+a$

$f'(c) = 0 \Rightarrow 3\left(2+\frac{1}{\sqrt{3}}\right)^2 + 2b\left(2+\frac{1}{\sqrt{3}}\right) + a = 0$ - - - - - - (i) 1

$f(1) = f(3) \Rightarrow$ b+a+6 = 32 + 9b +3a

or a + 4b = -13 - - - - - - - - - - - (ii) 1

Solving (i) and (ii) to get a=11, b= -6 1

19. Let 3x + 1 = A (–2x – 2) + B $\Rightarrow$ A = -3/2, B = –2 1

$I = \int \frac{-\frac{3}{2}(-2x-2)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{\left(\sqrt{6}\right)^2-(x+1)^2}}dx$ 1+1

$= -3\sqrt{5-2x-x^2} - 2.\ \sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + C$ 1

SECTION–C

20. (i) for all a, b ∈ A, (a, b) R (a, b), as a + b = b + a

∴ R is reflexive 1

(ii) for a, b, c, d ∈ A, let (a, b) R (c, d)

∴ a + d = b + c $\Rightarrow$ c + b = d + a $\Rightarrow$ (c, d) R (a, b)

∴ R is symmetric 1

(iii) for a, b, c, d, e, f, ∈ A, (a, b) R (c, d) and (c, d) R (e, f)

$\therefore a + d = b + c$ and $c + f = d + e$

$\Rightarrow a + d + c + f = b + c + d + e$ or $a + f = b + e$

$\Rightarrow (a, b)$ R (e, f) $\therefore$ R is Transitive 2

Hence R is an equivalence relation and equivalence class [(2, 5)] is ½

{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)} 1½

OR

Let $y \in S$, then $y=4x^2+12x+15$, for some $x \in N$

$\Rightarrow y = (2x + 3)^2 + 6 \Rightarrow x = \frac{(\sqrt{y-6})-3}{2}$, as $y > 6$ 1

Let $g : S \longrightarrow N$ is defined by $g(y) = \frac{(\sqrt{y-6})-3}{2}$ 1

$\therefore gof(x) = g(4x^2+12x+15) = g((2x+3)^2+6) = \frac{\sqrt{(2x+3)^2}-3}{2} = x$ 1

and $fog(y) = f\left(\frac{(\sqrt{y-6})-3}{2}\right) = \left[\frac{2\{(\sqrt{y-6})-3\}}{2} + 3\right]^2 + 6 = y$ 1

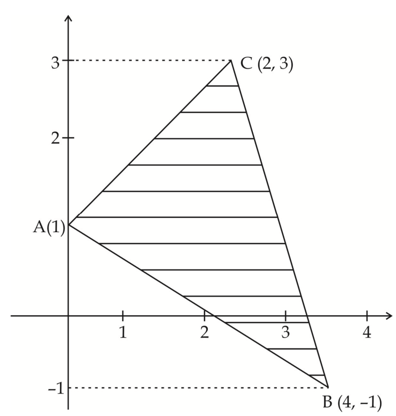
Hence $fog(y) = I_S$ and $gof(x) = I_N$

$\Rightarrow f$ is invertible and $f^{-1} = g$ 1

21. Let the lines be, AB: $x+2y = 2$, BC: $2x+y = 7$, AC = $y-x = 1$ 1

$\therefore$ Points of intersection are

A(0,1), B(4,-1) and C(2, 3) 1½



$A = \frac{1}{2}\int_{-1}^{3}(7 - y)\,dy - \int_{-1}^{1}(2 - 2y)\,dy - \int_{1}^{3}(y - 1)\,dy$ 1½

$= \frac{1}{2}\left(7y - \frac{y^2}{2}\right)_{-1}^{3} - (2y - y^2)_{-1}^{1} - \left(\frac{y^2}{2} - y\right)_{1}^{3}$ 1½

= 12 – 4 – 2 = 6sq.Unit. ½

22. Given differential equation is homogenous.

$\therefore$ Putting y = vx to get $\frac{dy}{dx}$ = v + $x\frac{dv}{dx}$ ½

$\frac{dy}{dx} = \frac{y\,sin\left(\frac{y}{x}\right) - xe^{y/x}}{x\,\sin\left(\frac{y}{x}\right)} \Longrightarrow \mathrm{v} + x\frac{dv}{dx} = \frac{v\,sinv - e^{v}}{sin\,v}$ 1

$\therefore \mathrm{v} + x\frac{dv}{dx} = \mathrm{v} - \frac{e^{v}}{sinv}$ or $x\frac{dv}{dx} = -\frac{e^{\mathrm{v}}}{sinv}$

$\therefore \int sinv\, e^{-v}\, dv = -\int \frac{dx}{x}\ or\ I_1 = -logx + c_1$ - - - - - - - - (i) 1

I_1 = sinv.e^{-v} + $\int \cos v\, e^{-v} dv$

= -sinv.e^{-v} – cosv $\mathrm{e}^{-\mathrm{v}} - \int \sin \mathrm{v}.\, \mathrm{e}^{-\mathrm{v}}$ dv

$I_1 = -\frac{1}{2}$ (sin v + cosv) $e^{-\mathrm{v}}$ 1

Putting (i), $\frac{1}{2}$ (sinv + cosv) $e^{-\mathrm{v}}$ = log$x$ +C_2

$\Rightarrow \left[\sin\left(\frac{\mathrm{y}}{\mathrm{x}}\right) + \cos\left(\frac{\mathrm{y}}{\mathrm{x}}\right)\right] e^{\frac{-y}{x}} = \log x^2 + \mathrm{C}$ 1

x = 1, y = 0 $\Rightarrow$ c = 1 1

Hence, Solution is $\left[\sin\left(\frac{\mathrm{y}}{\mathrm{x}}\right) + \cos\left(\frac{\mathrm{y}}{\mathrm{x}}\right)\right] e^{\frac{-y}{x}} = \log x^2 + 1$ ½

OR

$(x-a)^2 + (\mathrm{y}-\mathrm{b})^2 = r^2$ ....(i)

$\Rightarrow 2(x-a) + 2(\mathrm{y}-\mathrm{b})\frac{dy}{dx} = 0$ ....(ii) ½

$\Rightarrow 1 + (\mathrm{y}-\mathrm{b})\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ ....(iii) ½

$\therefore$ (y–b) = $-\frac{(1+y_1^2)}{y^2}$ 1½

From (ii), $(x-a) = \frac{y_1(1+y_1^2)}{y_2}$ 1½

Putting these values in (i)

$\frac{y_1^2(1+y_1^2)^2}{y_2^2}+\frac{(1+y_1^2)^2}{y_2^2}=r^2$ 1

or $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=r^2\left(\frac{d^2y}{dx^2}\right)^2$ 1

23. Here $\vec{a}_1=-3\hat{\imath}+\hat{\jmath}+5\hat{k}\,,\vec{b}_1=3\hat{\imath}+\hat{\jmath}+5\hat{k}$

$\vec{a}_2=-\hat{\imath}+2\hat{\jmath}+5\hat{k}\,,\vec{b}_2=-\hat{\imath}+2\hat{\jmath}+5\hat{k}$ ½

$(\vec{a}_2-\vec{a}_1).(\vec{b}_1\text{x}\,\vec{b}_2)=\begin{vmatrix}2&1&0\\-3&1&5\\-1&2&5\end{vmatrix}=2(-5)-1(-15+5)$ 1½

= –10 + 10 = 0

∴ lines are co–planer. ½

Perpendicular vector $(\vec{n})$ to the plane = $\overrightarrow{b_1}\text{x}\,\overrightarrow{b_2}$

$\begin{vmatrix}i&j&\hat{k}\\-3&1&5\\-1&2&5\end{vmatrix}=-5\hat{\imath}+10\hat{\jmath}-5\hat{k}$ 2

or $\hat{\imath}-2\hat{\jmath}+\hat{k}$ 2

∴Eqn. of plane is $\vec{r}.(\hat{\imath}-2\hat{\jmath}+\hat{k})=(\hat{\imath}-2\hat{\jmath}+\hat{k}).(-3\hat{\imath}+\hat{\jmath}+5\hat{k})=0$ 1½

or $x-2y+z=0$

24. Let E_1: Student resides in the hostel

E_2: Student resides outside the hostel

$P(E_1)=\frac{40}{100}=\frac{2}{5},P(E_2)=\frac{3}{5}$ ½+½

A: Getting A grade in the examination

$P\left(\frac{A}{E_1}\right)=\frac{50}{100}=\frac{1}{2}$ $P\left(\frac{A}{E_2}\right)=\frac{30}{100}=\frac{3}{10}$ 1+1

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P(\frac{A}{E_1})}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P(\frac{A}{E_2})}$$ 1

$$= \frac{\frac{2}{5}\cdot\frac{1}{2}}{\frac{2}{5}\cdot\frac{1}{2}+\frac{3}{5}\cdot\frac{3}{10}} = \frac{10}{19}$$ 1+1

25. Let the distance travelled @ 50 km/h be x km.

and that @ 80 km/h be y km.

∴ LPP is

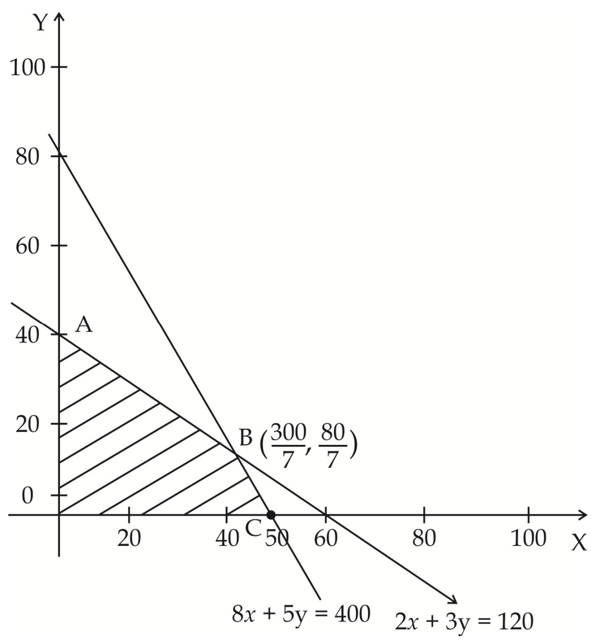
Maximize D = x + y

St. $2x + 3y \leq 120$ ½

$\frac{x}{50} + \frac{y}{80} \leq 1$ or $8x + 5y \leq 400$ 2

$x \geq 0$, $y \geq 0$

2



Vertices are.

$(0, 40)$, $\left(\frac{300}{7}, \frac{80}{7}\right)$, $(50,0)$

Max. D is at $\left(\frac{300}{7}, \frac{80}{7}\right)$

Max. D = $\frac{380}{7} = 54\frac{2}{7}$ km. 1½

26. Let P(x, y) be the position of the jet and the soldier is placed at A(3, 2)

$\Longrightarrow$ AP = $\sqrt{(x-3)^2 + (y-2)^2}$ ....(i) ½

As y = x^2 + 2 ⇒ y – 2 = x^2 ....(ii) ⇒ $AP^2 = (x-3)^2 + x^4$ = z (say) ½

$\frac{dz}{dx} = 2(x-3) + 4x^3$ and $\frac{d^2z}{dx^2} = 12x^2 + 2$ 2

$\frac{dz}{dx} = 0 \Rightarrow x = 1$ and $\frac{d^2z}{dx^2}$ (at x = 1) > 0 1+1

∴ z is minimum when x = 1, when x = 1, y = 1+2 = 3

∴ minimum distance = $\sqrt{(3-1)^2 + 1^2} = \sqrt{5}$ 1